Tracking of Lines in Spherical Images via Sub-Riemannian Geodesics on SO(3) [1]

Mashtakov A.P.

Program Systems Institute, Russian Academy of Sciences

alexey.mashtakov@gmail.com

In some imaging applications (e.g. in retinal imaging) it is natural to model the data by spherical images. In order to detect salient lines in these images we consider the problem of minimizing the functional $\int_0^l C(\gamma(s))\sqrt{\xi^2 + k_g^2(s)}\,\mathrm{d}s$ for a curve γ on a sphere with fixed boundary points and directions. The total length l is free, s denotes the spherical arclength, and k_g denotes the geodesic curvature of γ. Here the analytic external cost $C \geq \delta > 0$ is obtained from spherical data.

We lift this problem to the sub-Riemannian (SR) problem on Lie group $SO(3)$, and show that the spherical projection of certain SR-geodesics provides a solution to our curve optimization problem. In fact, this holds only for the geodesics whose spherical projection does not exhibit a cusp.

The problem is a spherical extension of a well-known contour perception model, where we extend the model by U. Boscain and F. Rossi to the general case $\xi > 0$, $C \neq 1$. For $C = 1$ we derive SR-geodesics and evaluate the first cusp time. We show that these curves have a simpler expression when they are parameterized by spherical arclength rather than by sub-Riemannian arclength. The case $C \neq 1$ (data-driven SR-geodesics) we solve via a SR Fast Marching method. Finally we show an experiment of vessel tracking in a spherical image of the retina, and study the effect of including the spherical geometry in analysis of vessels curvature.

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