Image Processing Based on Calculus of Variation and Partial Differential Equations

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Image Processing Based on Calculus of Variation and Partial Differential Equations
(*Outline*)

- *Image Denosing*
- *Image Inpainting*
- *Image Segmentation*
- *Image Decolorization*
- *Reference*

1 Image Denoising

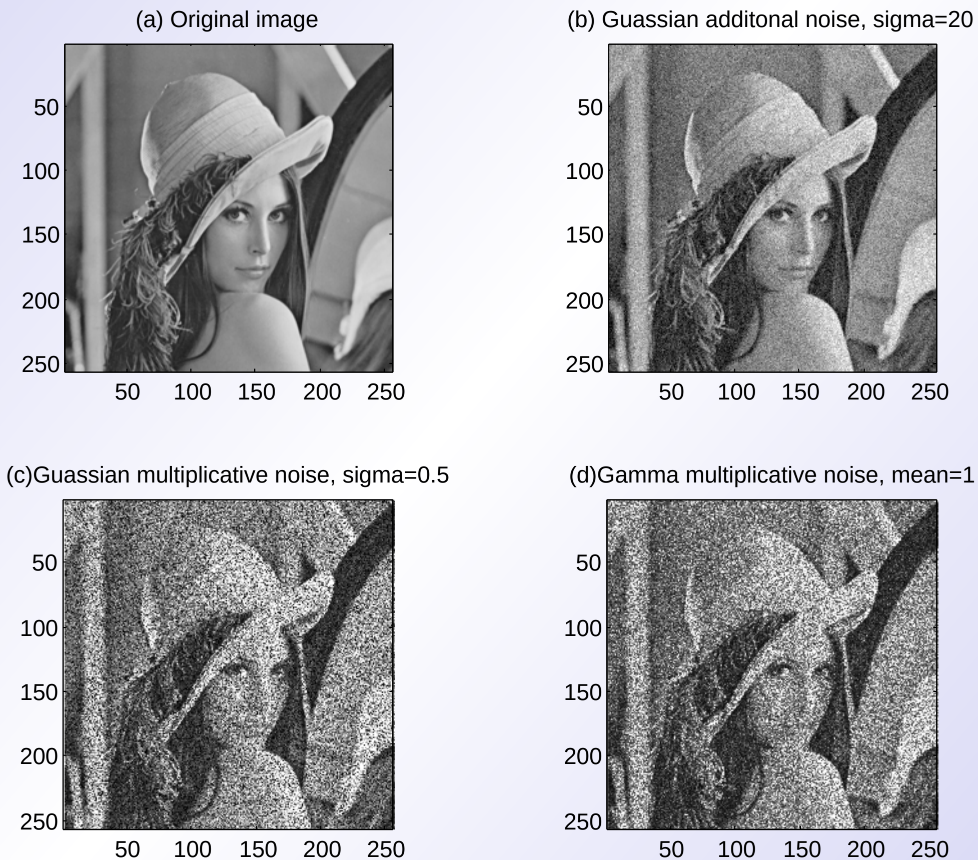
- Let $f : \Omega \to [0,1]$ be an observed image with noises, where $\Omega \subset \mathbb{R}^2$ is connected and bounded with Lipschitz boundary. Here u denotes the image to be recovered and n is the noise.
- Additive noise model:

$$f = u + n;$$

- Multiplicative noise model:

$$f = u \cdot n;$$

- Differences between additive noises and multiplicative noises：



1.1 Remove Additive Noise

Second order PDE Models

- Perona-Malik model [P. Perona and J. Malik, 1990]:

$$u_t = \nabla \cdot (g(|\nabla u|)\nabla u),$$

$$g(s) = \frac{1}{1 + (\frac{s}{K})^2} \tag{1.1}$$

and K is a threshold parameter.

- Total variation model [L. Rudin, S. Osher, and E. Fatemi, 1992]:

$$\inf_u \left\{ \int_\Omega |Du| + \lambda \int_\Omega |f-u|^2 \right\}$$

- **Drawbacks**: cause stair-case effect; lose some detailed information such as textures at the same time of denoising.

Fourth Order PDE Models

- Generalized Perona-Malik model [G. W. Wei, 1999] :

$$u_t + \nabla \cdot [g(\nabla u)\nabla \triangle u] = 0. \tag{1.2}$$

- TT model [J. Tumblin and G. Turk, 1999]:

$$u_t + \nabla \cdot [g(D^2 u)\nabla \triangle u] = 0. \tag{1.3}$$

- YK model [Y-L.You and M. Kaveh, 2000]:

$$u_t + \triangle[g(\triangle u)\triangle u] = 0. \tag{1.4}$$

- LLT model [M. Lysaker, A. Lundervold and X. C. Tai, 2003]:

$$\inf \left\{ \int \sqrt{|u_{xx}|^2 + |u_{xy}|^2 + |u_{yx}|^2 + |u_{yy}|^2} + \lambda \int |f-u|^2 \right\}$$

Our works

- Study the generalized Perona-Malik equation

$$u_t + \nabla \cdot [g(\nabla u)\nabla \triangle u] = 0.$$

  and get the existence and uniquness of the strong solutions to the generalized Perona-Malik equation in dimension one.([Z. JIN and X. Yang, 2010]).

- Study the following fourth order equation

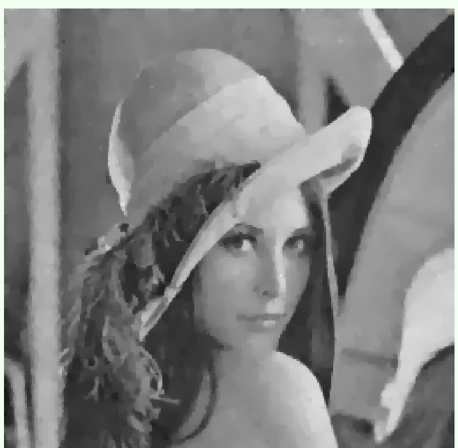
$$u_t + [g(u)u_{xx}]_{xx} = 0$$

  for image denoising, and get the global existence and uniquness of the solutions to this equation ([L. Min and X. Yang, 2014]).

Some Experimental Results for additive noise removal

(a) Original image

(b) Noisy image
SNR=16.48, sigma=20



(c) TV filter with adaptive parameter
SNR=28.54, Lamda=0.013

(d) LLT filter with adaptive parameter
SNR=29.56, Lamda=0.015

Figure: the denoising results. Top (left): original image. Top (right): noisy image. Bottom (left): denoised image by YK method. Bottom (right): denoised image by the proposed method.

1.2 Remove Multiplicative Noise

Some existing models

- RLO model [L. Rudin, P.L. Lions, and S. Osher,1993]-Gaussian multiplicative noise：

$$\inf_u \left\{ \int |Du| + \lambda_1 \int \frac{f}{u} + \lambda_2 \int \left(\frac{f}{u} - 1 \right)^2 \right\},$$

- Le model [Le T, Chartrand R, and Asaki T J, 2007]-Poisson noise:

$$\inf_u \left\{ \int |Du| + \lambda \int (u - f \log u) \right\}, \tag{1.5}$$

- AA model [G. Aubert and J.F. Aujol,2008]-Gamma multiplicative noise in SAR image :

$$\inf_u \left\{ \int |Du| + \lambda \int (\log u + \frac{f}{u}) \right\}, \tag{1.6}$$

Our works

- Propose the following model to remove the Raleigh multiplicative noise in ultrasound images([Z. Jin and X. Yang, 2011]) :

$$\inf_u \left\{ \int |Du| + \lambda \int \frac{(f-u)^2}{u} \right\}.$$

We prove the existence and uniqueness of the minimizer for the variational problem , and study the asymptotic behavior of the weak solution of the associated evolution equation.

- Propose another model to remove the Raleigh noise based on K-L distance([J. Huang and X. Yang, 2013]):

$$\inf_u \left\{ \int |Du| + \lambda \int \left(\frac{f}{\sqrt{u}} \log \frac{f}{u} - \frac{f}{\sqrt{u}} + \sqrt{u} \right) \right\},$$

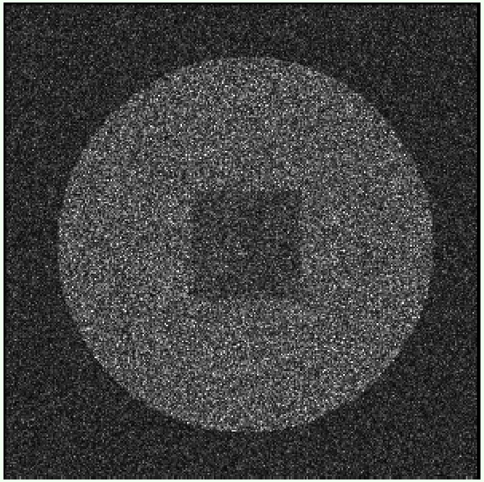
and give a fast numerical algorithm to solve this model.

Some Experimental Results for multiplicative noise removal

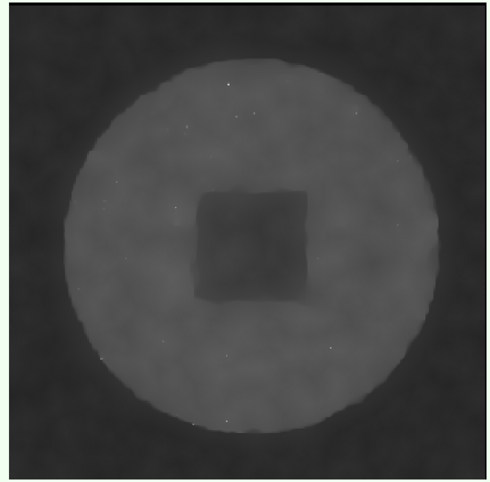
- Multiplicative noise removal for synthetic images

(a1) corrupted image with poisson noise

(a2) Le model



(b1) corrupted image with Gamma noise



(b2) AA model

(c1) corrupted image with Raleigh noise

(c2) JY model

- Multiplicative noise removal for real ultrasonic images

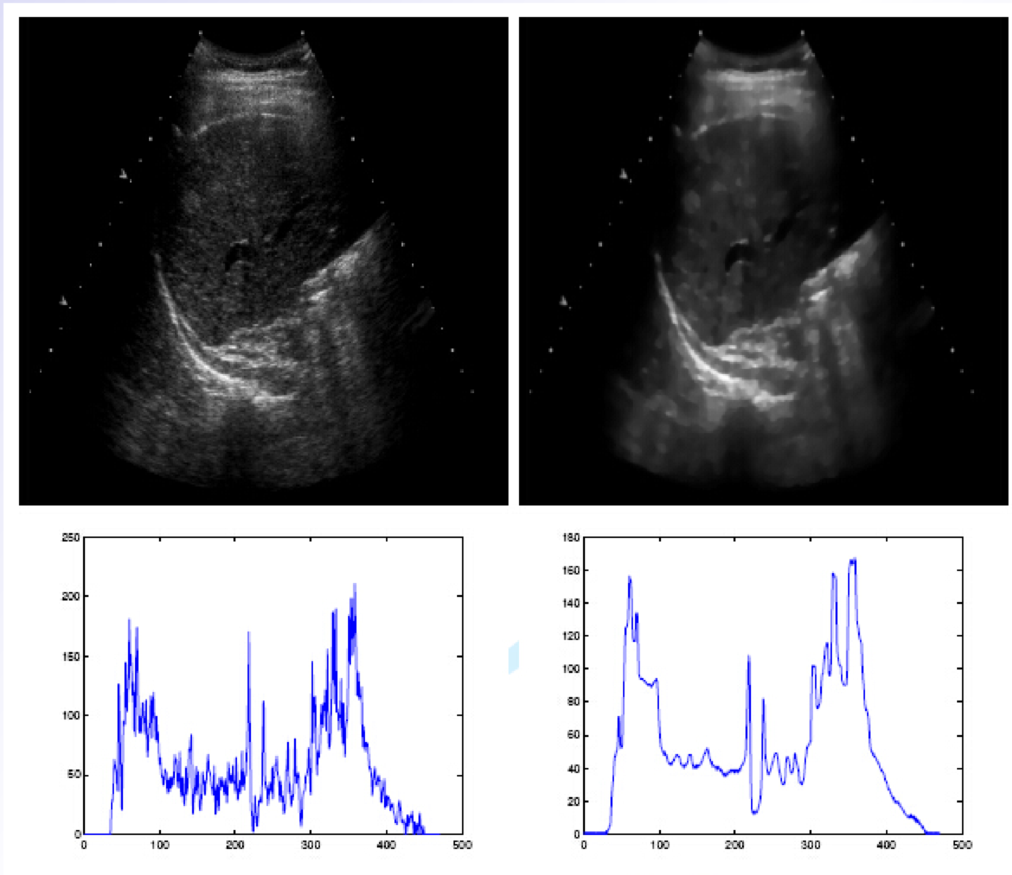


Figure: Speckle reduction of a real ultrasound abdomen image. Top Left: the real ultrasound abdomen image; Top Right: the restored image by proposed method; BottomLeft: the signal of column 320 of the real abdomen ultrasound image; Bottom Right: the signal of column 320 of the restored image by the proposed method.

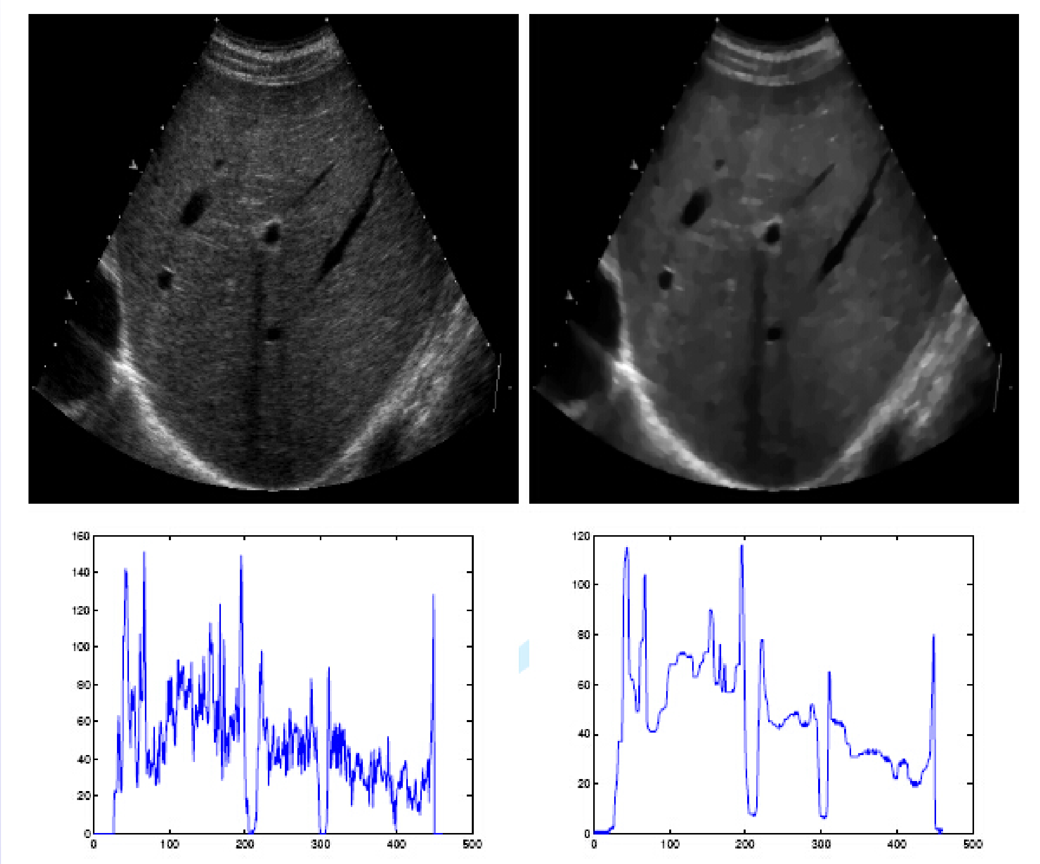
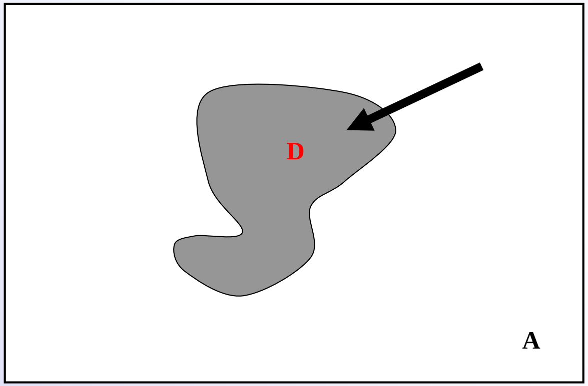


Figure: Speckle reduction of a real ultrasound abdomen image. Top Left: the real ultrasound abdomen image; Top Right: the restored image by proposed method; BottomLeft: the signal of column 320 of the real abdomen ultrasound image; Bottom Right: the signal of column 320 of the restored image by the proposed method.

2 Image Inpainting

- Image inpainting is the processing of restoring regions of missing information in digital images by using the information surrounding these regions.
- Let $f : \Omega \setminus D \to [0,1]$ be a damaged image, where $D \subset\subset \Omega \subset \mathbb{R}^2$ is the inpainting damain. Here u denotes the image to be inpainted.



2.1 Inpainting Models

- BSCB model [Bertalmio et. al., 2000] :

$$u_t = \nabla^{\perp} u \cdot \nabla \triangle u,$$

where $\nabla^{\perp}$ denotes perpendicular gradient $(-\partial_y, \partial_x)$. Actually BSCB model is a transport equation of the third-order that convects image Laplacian $(\triangle u)$ along isophote direction $(\nabla^{\perp} u)$.

- Modified BSCB model [Bertalmio et. al., 2001] :

$$\begin{cases} w_t + \nabla^{\perp} u \cdot \nabla w = \nu \triangle w, \\ \triangle u = w, \end{cases}$$

where $\nu > 0$ is the viscosity coefficient.

- CDD model [T.Chan et. al., 2001]:

$$u_t = \nabla \cdot [\frac{G(k,x)}{|\nabla u|}\nabla u] + \lambda(x)(f-u),$$

where $k = div(\frac{\nabla u}{|\nabla u|})$ is the the curvature of the isophote line.

- Euler's elastica inpainting model [T.F. Chan, S.H. Kang, and J. Shen, 2002]:

$$\inf_u \left\{ \int_\Omega \left(a + b \left(\nabla \cdot \frac{\nabla u}{|\nabla u|} \right)^2 \right) + \frac{\lambda}{2} \int_{\Omega \setminus D} (f-u)^2 \right\},$$

where $a, b > 0$ and $\lambda > 0$ is the weighted parameter.

2.2 Our work

- Following the Modified BSCB model, we consider the following parabolic-elliptic system for image inpainting ([Z. Jin and X. Yang, 2010]):

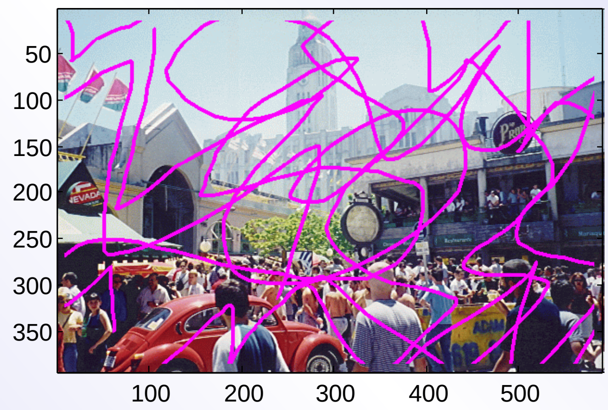
$$\begin{cases} w_t + \nabla^{\perp} I \cdot \nabla w = \nu\, div[g(|\nabla G_\sigma * w|)\nabla w], \\ \triangle I = w, \end{cases} \tag{2.1}$$

and and establish the existence and uniqueness of weak solutions to the system in dimension two.

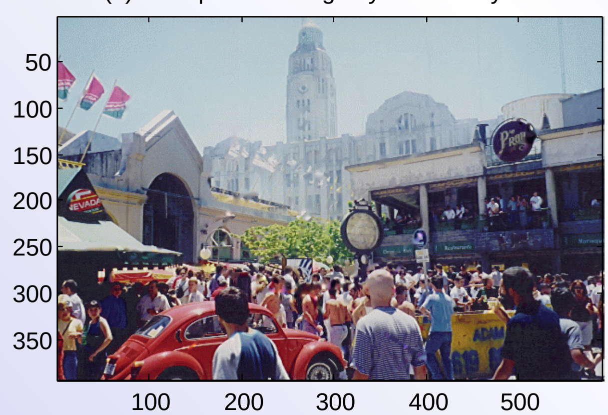
Some Experimental Results for Image Inpainting

- Modified BSCB inpainting:

(a) the damaged image



(b) the inpainted image by MBSCB system



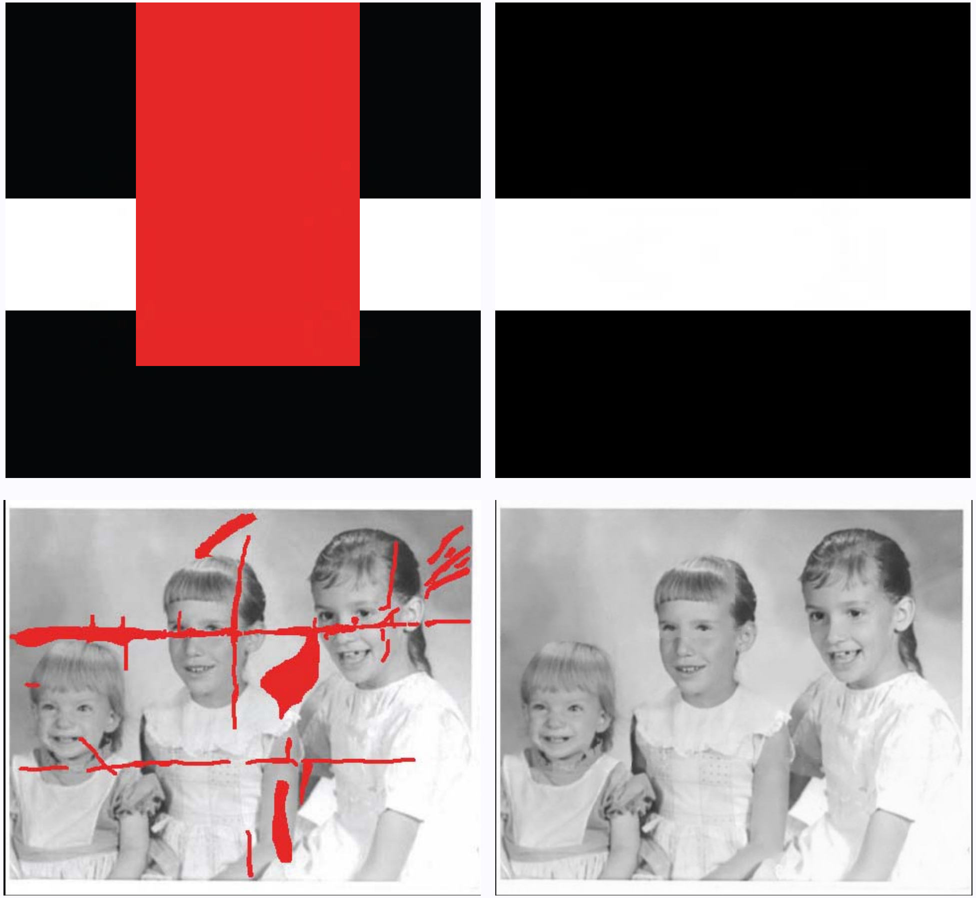
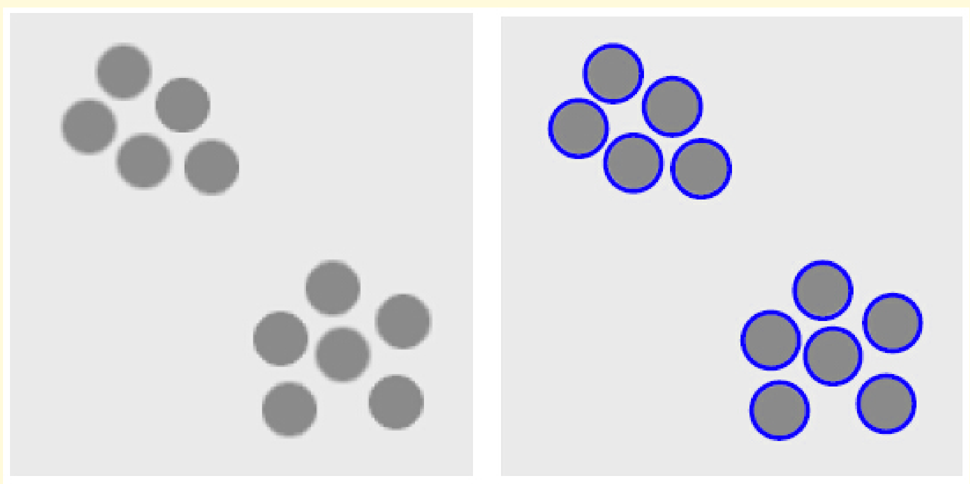


Figure: the inpainting results. Left: damaged images. Right: inpainted images.

3 Image Segmentation

- Image segmentation is the process of partitioning a digital image into multiple segments (sets of pixels, also known as superpixels). The goal of segmentation is to divide the image into regions that belongs to distinct objects in the depicted scene.
- Many segmentation methods rely heavily in some way on edge detection. Let f denote the given grayscale image on a domain Ω to be segmented. Let C be an edge set curve of the distinct objects in f.



(a) Input image (b) Segmented image with edges

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3.1. Some Existing Models

- Mumford-Shah model [D. Mumford and J. Shah, 1989]:

$$\min_{u,C}\left\{\mu\text{Length}(C)+\lambda\int_{\Omega}(f(x)-u(x))^2dx+\int_{\Omega\backslash C}|\nabla u(x)|^2dx\right\},$$

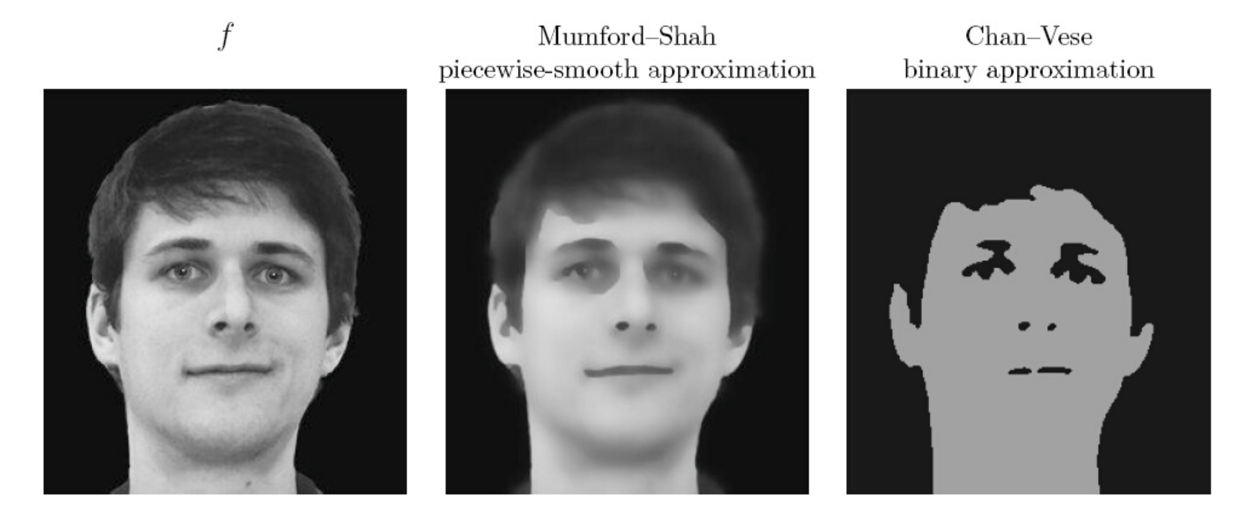
where the first term ensures regularity of C, the second term encourages u to be close to f, and the third term ensures that u is differentiable on $\Omega\setminus C$. The Mumford–Shah model suggests selecting this edge set C as the segmentation boundary.

- Chan-Vese model [T.F. Chan, L.A. Vese, 2001]:

$$\min_{c_1,c_2,C}\left\{\mu\text{Length}(C)+\nu\text{Area}(inside(c))+\lambda_1\int_{inside(C)}(f(x)-C_1)^2dx\right.$$
$$\left.+\lambda_2\int_{outside(C)}|f(x)-C_2|^2dx\right\},$$

where C is the boundary of a closed set and C_1, C_2 are the values of u respectively inside and outside of C. The first term controls the regularity by penalizing the length. The second term penalizes the enclosed area of C to control its size. The third and fourth terms penalize discrepancy between the piecewise constant model u and the input image f.

- Image segmentation using Mumford-Shah model and Chan-Vese model:



3.2 Our Works

- By consider the speckle noise with Fisher-Tippet distribution:

$$p(f) = 2e^{(2f(x,y) - \ln 2\sigma^2 - e^{2f(x,y) - \ln 2\sigma^2})},$$

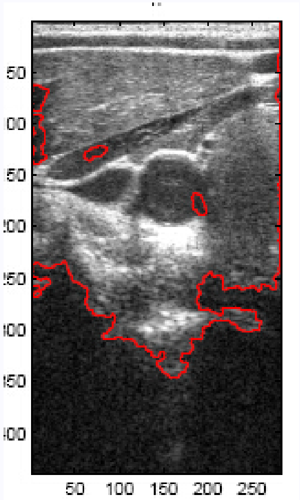
we propose the following variational model to segment the ultrasound images([J. Huang, X. Yang and Y. Chen, 2011]):

$$\min_{\sigma_i,\sigma_o,C} \left\{ \mu \text{Length}(C) - \int_{inside(C)} \log p(f(x,y;\sigma_i))dxdy - \int_{outside(C)} \log p(f(x,y;\sigma_o))dxdy \right\},$$

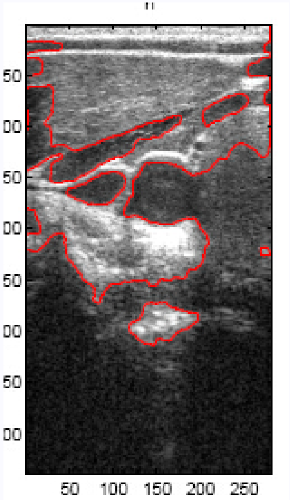
and give a fast numerical algorithm by incorporating PDHG method to solve the relaxed model.

- Ultrasound image segmentation using Huang-Yang-Chen model:

(a) real ultrasound image



(c) rayleigh segmentation model



(d) the proposed model

Image Denoising
Image Inpainting
Image...
Image...
Reference

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- By consider the speckle noise with F-T distribution and a prior super-ellipse shape of the kidney, we propose another variational model to segment the ultrasound kidney images([J. Huang, X. Yang, Y. Chen and L. Tang, 2013]):

$$\min_{\sigma_i,\sigma_o,C} \Big\{ \int_\Omega |\nabla H(\phi)| + \mu_1 \int_\Omega L_1(\sigma_i^2, I) H(\phi) + L_2(\sigma_o^2, I)(1 - H(\phi)) dxdy \Big\},$$

such that

$$\int_\Omega (H(\phi) - H(\tilde{\phi}))^2 dxdy < k^2.$$

where $\int_\Omega |\nabla H(\phi)|$ is the boundary length of the segmented region, k^2 is a positive parameter, which determines the maximum error of two shapes, and μ_1 is the positive weighted parameter, which keeps a balance between boundary smoothness and region force.

- Also, we develop a fast algorithm to solve numerically the above model by incorporating the spit Bregman technique.

• Ultrasound kidney segmentation using Huang-Yang-Chen-Tang model:

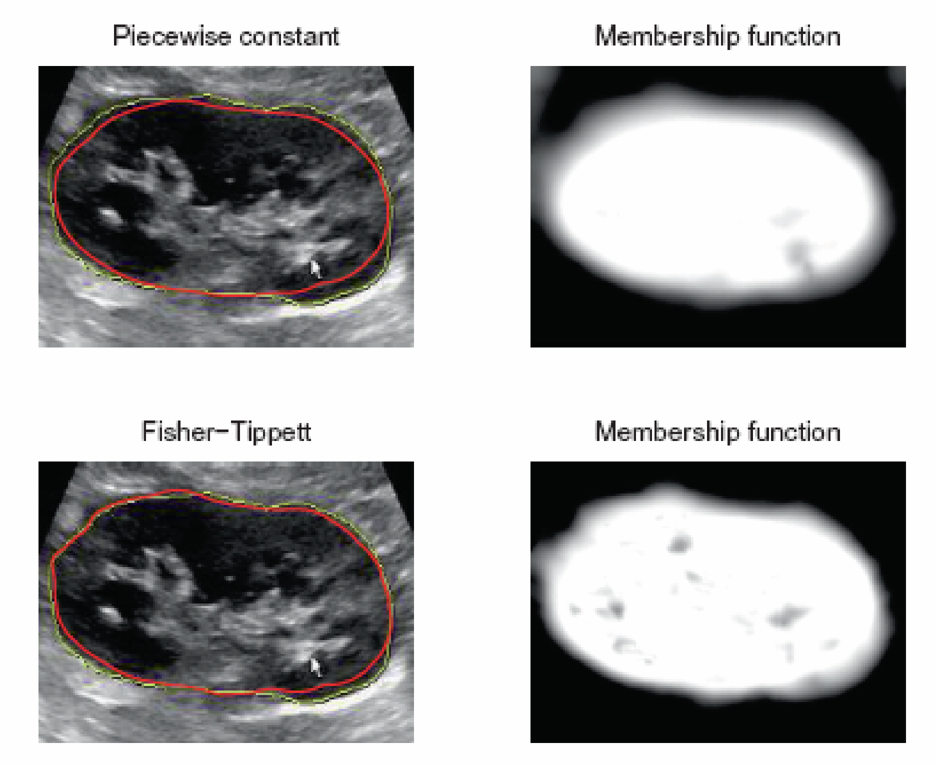

Figure: yellow line is delineated by a radiologist, and red line is the segmentation result. (Top) Left: segmentation results without shape constraint using piecewise constant method. (Bottom) Left: segmentation result of our method

Our recent works

- Renal Lesion Segmentation of Medical Ultrasound Images based on Dempster-Shafer Evidence Theory[L. Gui and X. Yang, preprint].

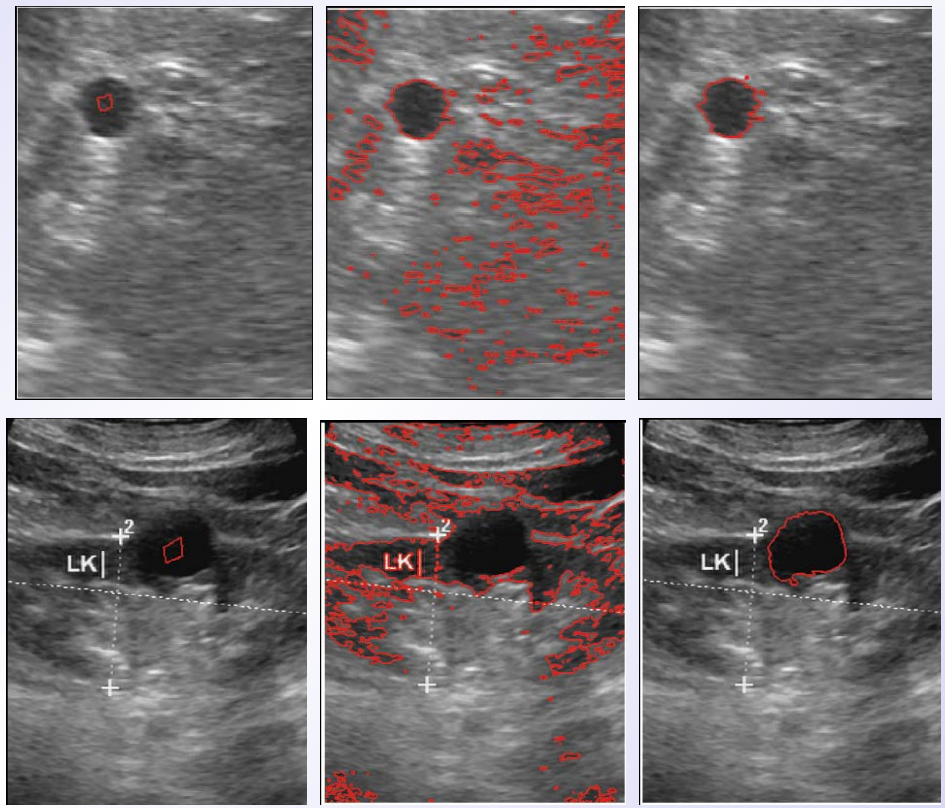

Figure: Renal Lesion Segmentation of ultrasound images. Left column: initial contours; middle column: CV model; right column: our method.

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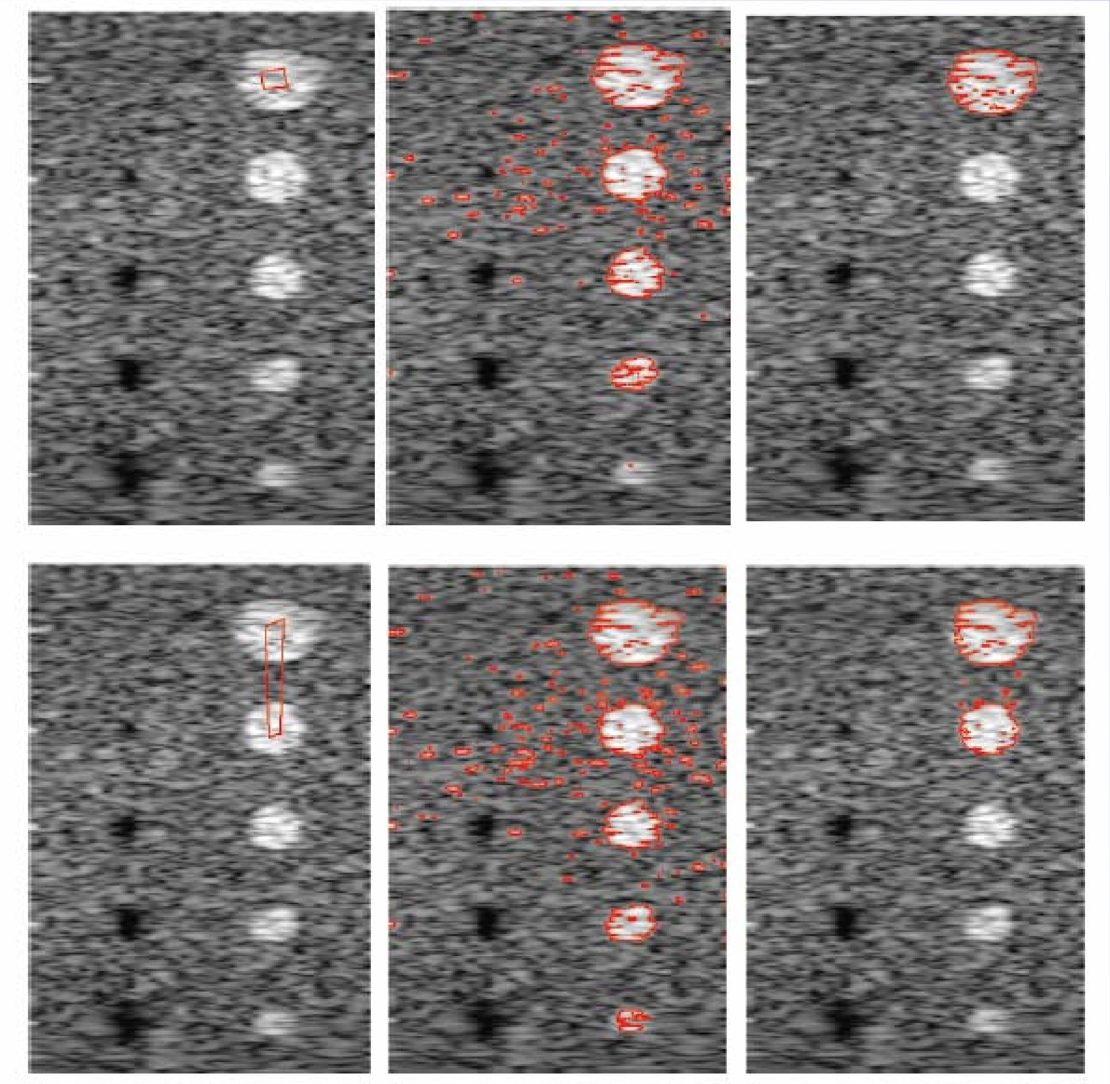


Figure: Segmentation of synthetic images. Left column: initial contours; middle column: CV model; right column: our method.

- Ultrasound prostate image segmentation based on locally region active contour with edge profile[X. Li and X. Yang, preprint].

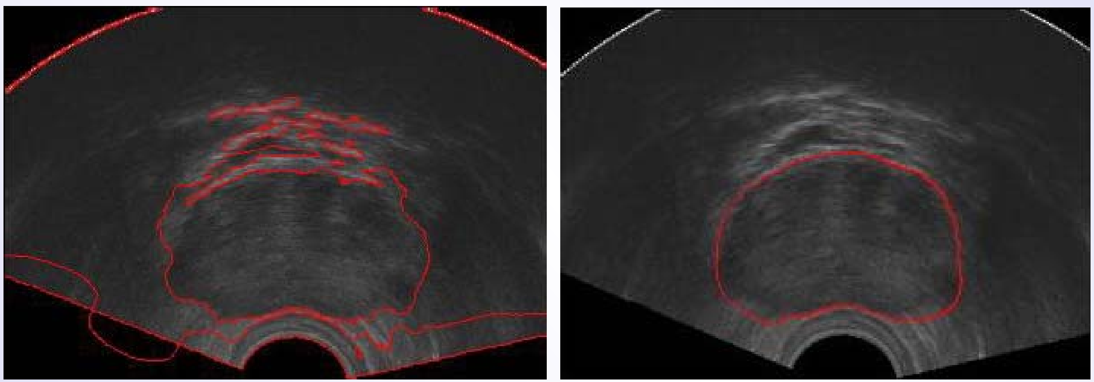


Figure: utrasound prostate image segmentation. Left: segmentation result of the Li's method; Right: segmentation result of our algorithm with edge descriptor.

4 Image Decolorization

- Image decolorization is the process to convert a color image to a grayscale one, which is a basic tool in digital printing, photograph rendering and single-channel image processing.

- **Many interesting and important applications**:

  black-and-white printing

  single channel image processing

  non-photorealistic rendering with black and white media

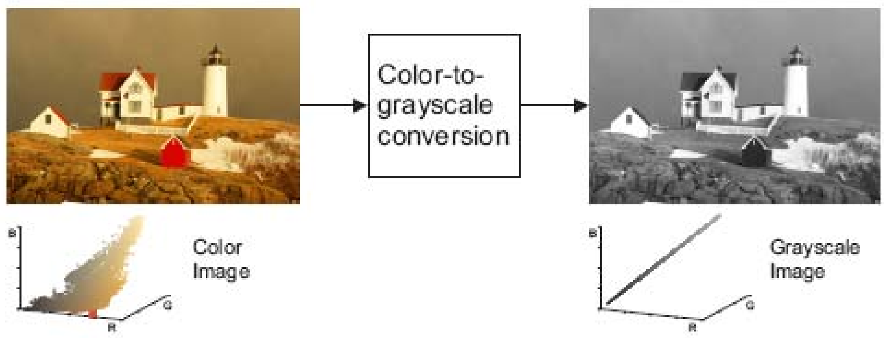


Figure 1: *The color to grayscale image conversion.*

The main challenge of this research is to preserve as much information from the original color image as possible, and to generate a perceptually plausible grayscale image.

4.1 Existing methods for image decolorization

- **One simple approach:** use the lightness channel of CIELab only, and generate a grayscale image.
- **global mapping methods:**
  rgb2gray
  Grundland's method[M. Grundland and N. A. Dodgson, 2007]
  Qiu's method[M. Qiu, G. Finlayson, and G. Qiu, 2008]
  Lu's method[Cewu Lu, Li Xu, Jiaya Jia, 2012] et al.
- **local mapping methods:**
  Smith's method[K. Smith, P.E. Asndes and K.M. Jöelle Thollot, 2008]
  Bala's method[R. Bala and R. Eschbach, 2004] et al.
- **Remark:** In 2008, Cadik pointed out that Smith's and Grundland's methods are effective methods for image decolorization based on a paired comparison paradigm. It is also reported that the performance of Lu's method is better than the other existing methods.

4.2 The proposed global model [Z. Jin and Michael Ng, 2014]

- Let $I(x) = \{R(x), G(x), B(x)\}$ be a color image defined on an open, bounded domain $\Omega \subset \mathbb{R}^2$. Let $\mathbf{m} = (m_r, m_g, m_b)$ be the mean vector containing the red, green and blue color channels of the input color image. Denote by

$$\mathbf{K} = \begin{pmatrix} k_{r,r} & k_{r,g} & k_{r,b} \\ k_{g,r} & k_{g,g} & k_{g,b} \\ k_{b,r} & k_{b,g} & k_{g,g} \end{pmatrix}$$

  the covariance matrix of the red, green and blue channels of the input color image. The desired grayscale image L according to the following mapping:

$$L(x) = \alpha R(x) + \beta G(x) + \gamma B(x), \quad \forall x \in \Omega, \qquad (4.1)$$

- **Qiu's model:** the following optimization problem is considered to solve $\mathbf{u}$:

$$\min_{\mathbf{u}} \left\{ -\frac{\mathbf{uKu}^T}{2} + \frac{\lambda}{2}(\mathbf{mu}^T - 1)^2 \right\} \qquad (4.2)$$

  subject to

$$\alpha + \beta + \gamma = 1, \;\; 0 \leq \alpha, \beta, \gamma \leq 1. \qquad (4.3)$$

♣ Our global model:

- By introducing the regularization term $\frac{\mu}{2}\mathbf{u}\mathbf{u}^T$ to ensure that the objective function is strictly convex, we consider the following optimization problem to solve $\mathbf{u}$:

$$\min_{\mathbf{u}} \left\{ -\frac{\mathbf{u}\mathbf{K}\mathbf{u}^T}{2} + \frac{\lambda}{2}(\mathbf{m}\mathbf{u}^T - 1)^2 + \frac{\mu}{2}\mathbf{u}\mathbf{u}^T \right\} \tag{4.4}$$

subject to

$$\alpha + \beta + \gamma = 1,\ \ 0 \leq \alpha, \beta, \gamma \leq 1. \tag{4.5}$$

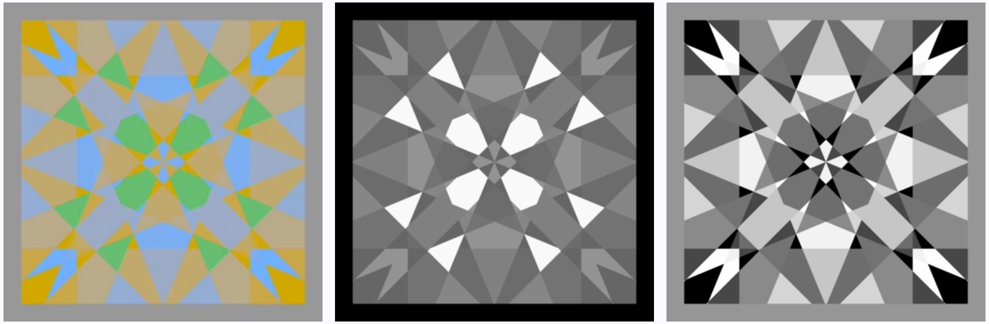
- The main computational tasks of the proposed global model:

(i) calculate the mean vector $\mathbf{m}$ and covariance matrix $\mathbf{K}$;

(ii) compute the eigenvalues of $\mathbf{A}$ to determine the value of μ;

(iii) solve the convex optimization problem by the MATLAB routine: QUADPROG

• One example of decolorization by our global model



(a) (b) (c)

Figure 1: (a) input color image; (b) grayscale image by Qiu's method; (c) grayscale image by proposed global method

Parameter	Eigenvalues of Hessian	Solution
$\lambda = 21$	-0.1114, -0.0115, 24.3257	(0,1,0)
$\lambda = 180$	-0.1113, -0.0115, 208.6377	(0,1,0)
$\lambda = 650$	-0.1113, -0.0115, 753.4592	(0,1,0)

Table 1: The eigenvalues of Hessian of the objective function in Qiu's model and the corresponding solutions, where **K** and **m** is computed based on the color image in Figure 1(a).

.

4.3 The proposed local model [Z. Jin, F. Li and Michael Ng, 2014]

- In order to preserve more contrast details, by employing a local transformation $\mathbf{u}(x) = (\alpha(x), \beta(x), \gamma(x))(\forall x \in \Omega)$, we design a local mapping to get the decolorized grayscale image

$$L(x) = \alpha(x)R(x) + \beta(x)G(x) + \gamma(x)B(x).$$

- At a pixel location x based on its neighborhood $N_w(x)$, denote by

$$\mathbf{m}(\mathbf{x}) := (m_r(x), m_g(x), m_b(x)),$$

$$\mathbf{K}(x) := \begin{pmatrix} k_{r,r}(x) & k_{r,g}(x) & k_{r,b}(x) \\ k_{g,r}(x) & k_{g,g}(x) & k_{g,b}(x) \\ k_{b,r}(x) & k_{b,g}(x) & k_{g,g}(x) \end{pmatrix}, \quad \forall\, x \in \Omega,$$

the local mean value and the local covariance matrix.

- Note that at a pixel location x the local variance of $L(x)$ is given by

$$\mathbf{Var}_w(L(x)) = \mathbf{u}(x)\mathbf{K}(x)\mathbf{u}^T(x).$$

- In order to minimize the differences among the local transformations at the nearby pixel locations, the total variation regularization $\int_\Omega |D\mathbf{u}|$ is also incorporated in the functional. We consider the following variational problem:

$$\min_{\mathbf{u}\in\Lambda}\left(\int_\Omega |D\mathbf{u}(x)|\mathrm{dx} - \frac{\tau}{2}\int_\Omega \mathbf{u}(x)\mathbf{K}(x)\mathbf{u}(x)^T\mathrm{dx} + \frac{\lambda}{2}\int_\Omega (\mathbf{m}(\mathbf{x})\mathbf{u}(\mathbf{x})^T - 1)^2\mathrm{dx} + \frac{\mu}{2}\int_\Omega |\mathbf{u}(\mathbf{x})|^2\mathrm{dx}\right) \tag{4.6}$$

where

$$\begin{aligned}\Lambda \;:=\; \{&\mathbf{u}(x) = (\alpha(x), \beta(x), \gamma(x))|\mathbf{u}(x)\in BV(\Omega;\mathbb{R}^3),\\ &\alpha(x)+\beta(x)+\gamma(x) = 1, 0 \le \alpha(x), \beta(x), \gamma(x) \le 1\}.\end{aligned} \tag{4.7}$$

and τ, λ and μ are positive numbers to balance the four terms in the above objective function.

♣ One example of decolorization by our local model

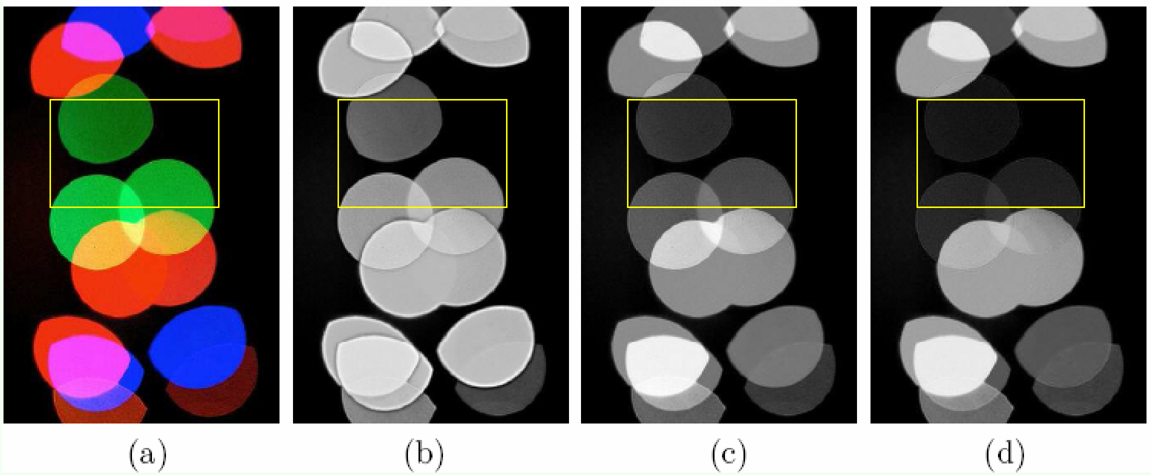


Figure 3.1: (a) Input color image; (b) the proposed method; (c) Qiu's method; (d) Grundland's method.

-

- Let us consider the covariance matrix of the input color image Figure 3.1(a):

$$\mathbf{K} = \begin{pmatrix} 0.1509 & 0.0306 & 0.0351 \\ 0.0306 & 0.0554 & 0.0215 \\ 0.0351 & 0.0215 & 0.1034 \end{pmatrix}.$$

It is clear that the covariance information related to green channel are relatively low.

♣ Theoretical Analysis and Numerical Algorithm

- The existence of the variational model:

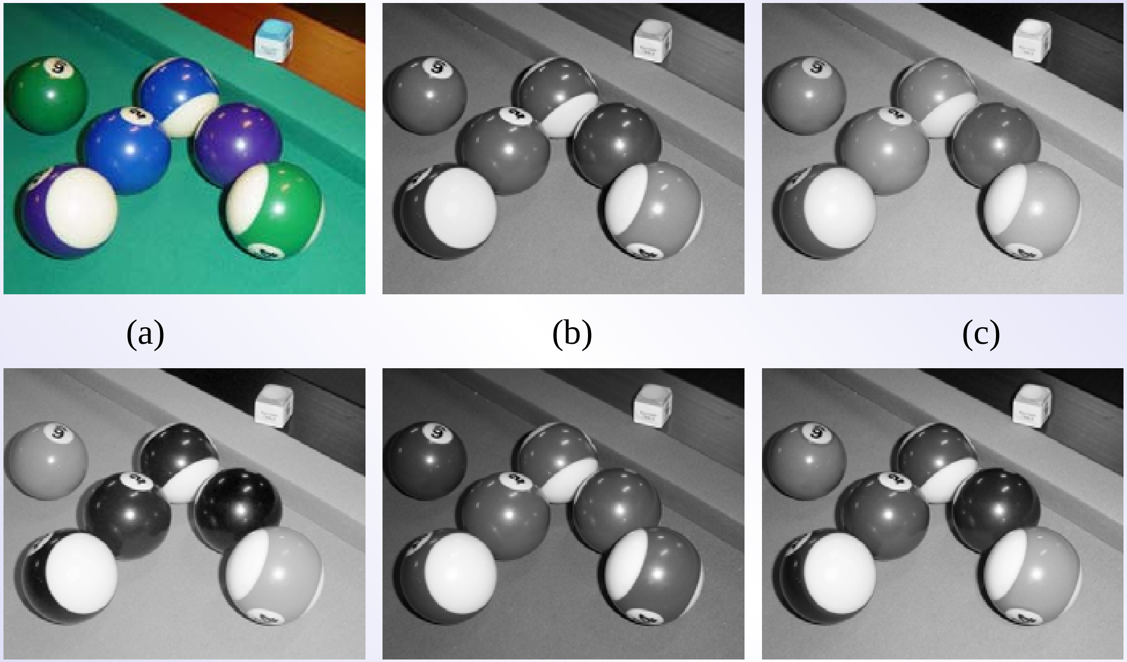
4.1 *Let* $I \in L^{\infty}(\Omega; \mathbb{R}^3)$ *and*

$$\mu > \max\{\tau C_1 + \lambda M_1, \tau C_2 + \lambda M_2, \tau C_3 + \lambda M_3\}.$$

Then the variational problem (4.6)-(4.7) admits a unique solution.

- A fast alternating minimization algorithm is designed to numerically solve the proposed local model and its convergence is obtained.

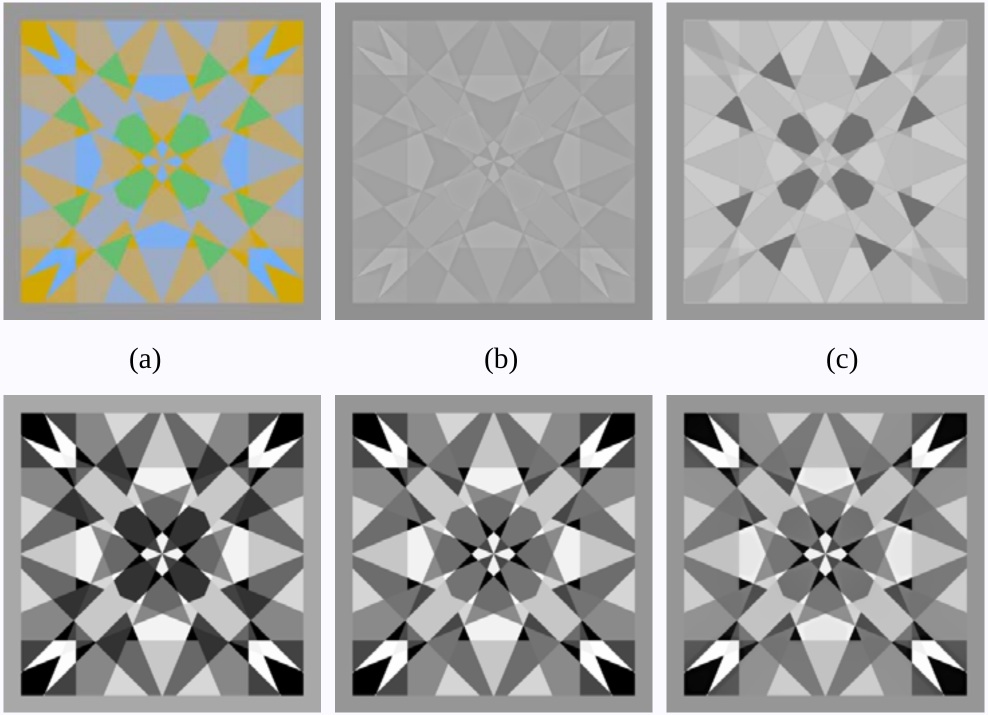
Grayscale images obtained by different methods



(a) (b) (c)

(d) (e) (f)

Figure 4.2: (a) input color Images; (b)Smith's method; (c) Grundland's method; (d) Lu's method; (e) the proposed global method; (f) the proposed local method



(a) (b) (c)

(d) (e) (f)

Figure 4.4: (a) input color Images; (b)Smith's method; (c) Grundland's method; (d) Lu's method; (e) the proposed global method; (f) the proposed local method

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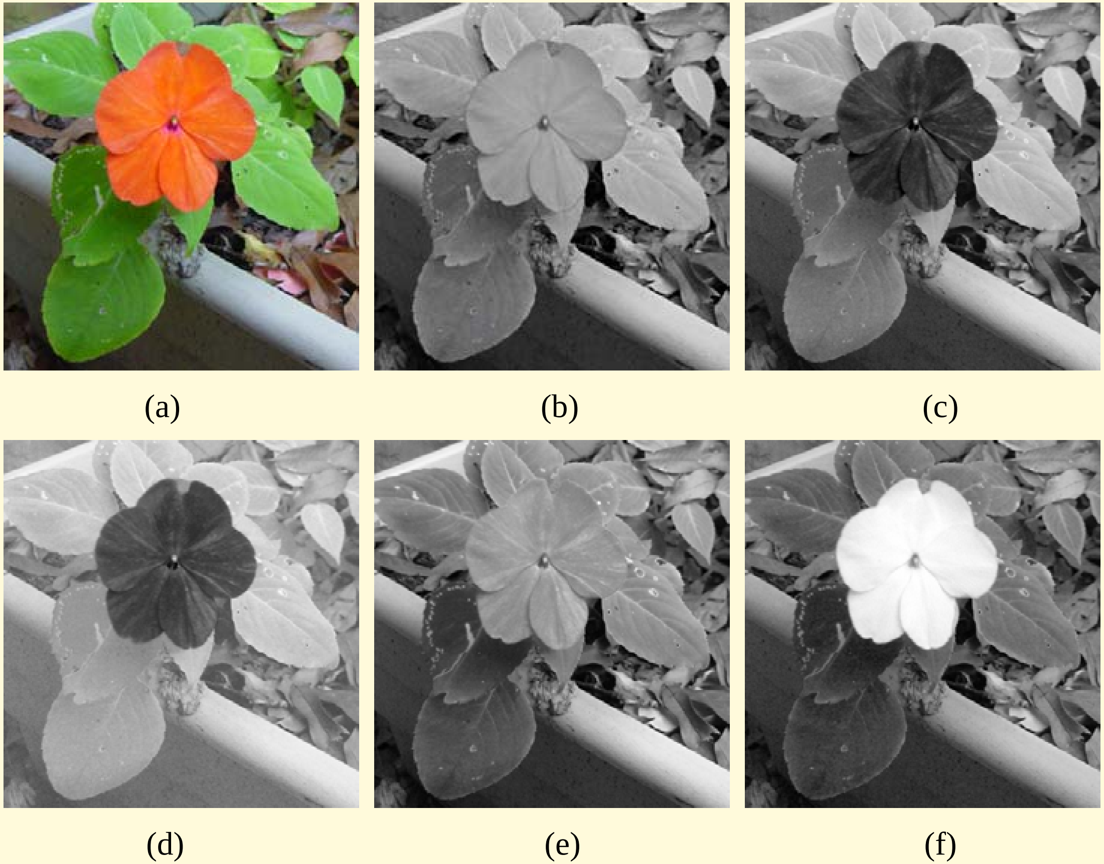
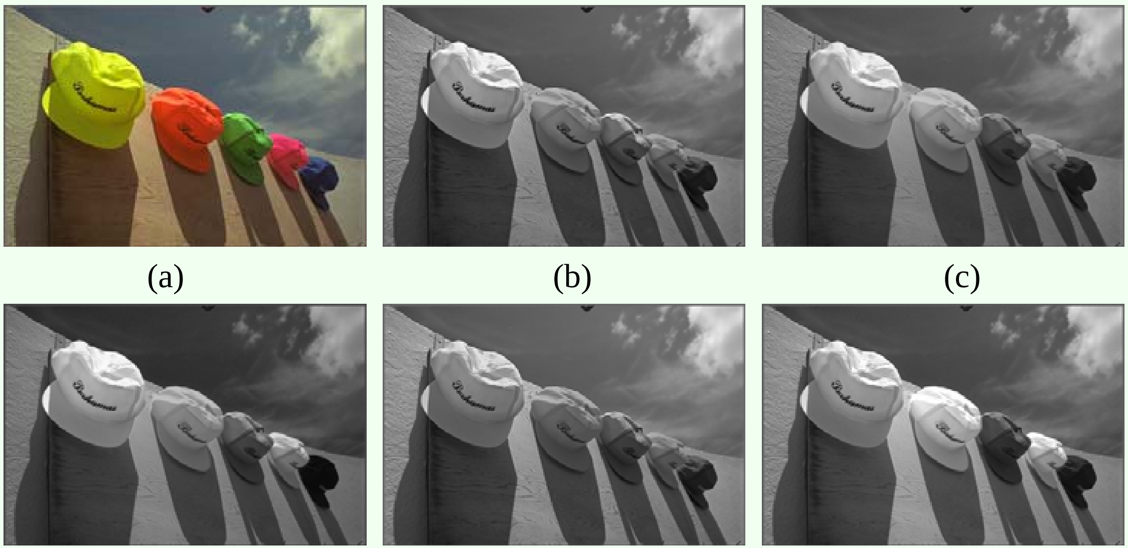


Figure 4.5: (a) input color Images; (b)Smith's method; (c) Grundland's method; (d) Lu's method; (e) the proposed global method; (f) the proposed local method



(a) (b) (c)

(d) (e) (f)

Figure 4.6: (a) input color Images; (b)Smith's method; (c) Grundland's method; (d) Lu's method; (e) the proposed global method; (f) the proposed local method

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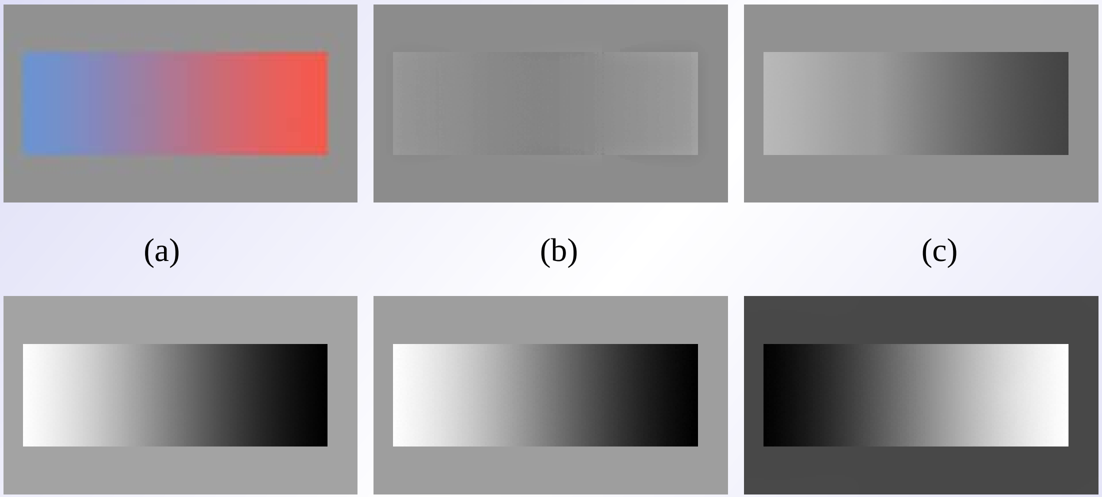
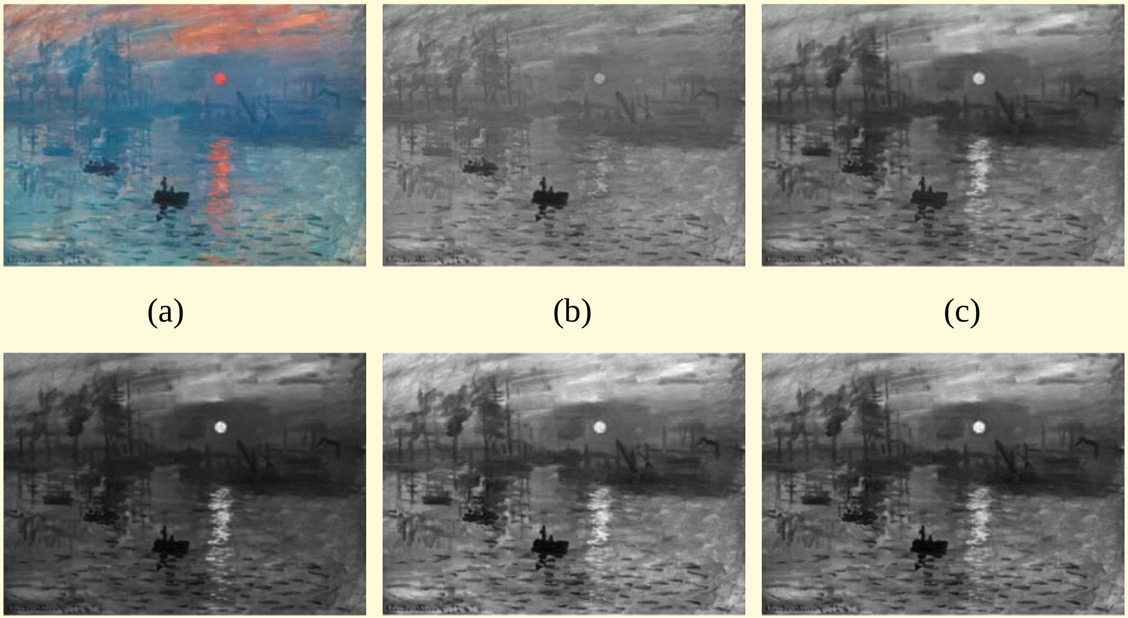


(a) (b) (c)

(d) (e) (f)

Figure 4.7: (a) input color Images; (b)Smith's method; (c) Grundland's method; (d) Lu's method; (e) the proposed global method; (f) the proposed local method



(a) (b) (c)

(d) (e) (f)

Figure 4.8: (a) input color Images; (b)Smith's method; (c) Grundland's method; (d) Lu's method; (e) the proposed global method; (f) the proposed local method

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Thank you!